New Vacuum Solutions for Quadratic Metric-affine Gravity

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Mathematical model

$$\text{Spacetime} = \{M, g, \Gamma\}$$

Here M is a real 4-manifold, g a Lorentzian metric and Γ is an affine connection, i.e.

$$\nabla_\mu u^\lambda = \partial_\mu u^\lambda + \Gamma^\lambda{}_{\mu\nu} u^\nu$$

The action is

$$S := \int q(R)$$

where $q(R)$ is a Lorentz invariant purely quadratic form on curvature.

The field equations are

$$\partial S/\partial g = 0, \tag{1}$$

$$\partial S/\partial \Gamma = 0. \tag{2}$$

Known solutions

Definition 1 *We call a spacetime* $\{M, g, \Gamma\}$ Riemannian *if the connection is Levi-Civita (i.e.* $\Gamma^{\lambda}{}_{\mu\nu} = \left\{ {\lambda \atop \mu\nu} \right\}$*).*

Following known solutions:

- Einstein spaces
- PP-waves with parallel Ricci curvature
- Certain explicitly given torsion waves

Generalised pp-waves

Definition 2 *A pp-wave is a Riemannian spacetime which admits a non-vanishing parallel spinor field* ($\nabla\chi = 0$).

Consider plane-wave solutions of polarized Maxwell equation

$$*\mathrm{d}A = \pm\mathrm{d}A.$$

Definition 3 *A generalised pp-wave is a metric compatible spacetime with pp-metric and torsion*

$$T := \frac{1}{2}\,\mathrm{Re}(A \otimes dA)$$

Main result

Theorem 1 *Generalised pp-waves of parallel Ricci curvature are solutions of the field equations (1), (2).*

Proof is done by 'brute force'. We write down the field equations (1), (2) for general metric compatible spacetimes and substitute formulae for torsion, Ricci and Weyl into these. With $\nabla Ric = 0$, we get the result.

Result can be found in "PP-waves with torsion and metric affine gravity", 2005 V. Pasic, D. Vassiliev, *Class. Quantum Grav. 22 3961-3975*.

Interpretation

- Curvature of generalised pp-waves is split.

- Torsion and torsion generated curvature are waves traveling at the speed of light.

- Underlying pp-space can be viewed as the 'gravitational imprint' created by wave of some massless field.

- Mathematical model for neutrino?

Metric-affine model for neutrino

Neutrino field in metric compatible spacetime described by

$$S_{\text{neutrino}} := 2i \int \left(\xi^a \, \sigma^\mu{}_{a\dot{b}} \, (\nabla_\mu \bar{\xi}^{\dot{b}}) - (\nabla_\mu \xi^a) \, \sigma^\mu{}_{a\dot{b}} \, \bar{\xi}^{\dot{b}} \right),$$

In generalised pp-space Weyl's equation takes form

$$\sigma^\mu{}_{a\dot{b}} \{\nabla\}_\mu \, \xi^a = 0.$$

Constructed pp-wave type solutions of Einstein-Weyl model

$$S_{\text{EW}} := \int \mathcal{R} + S_{\text{neutrino}},$$

$$\partial S_{\text{EW}} / \partial g = 0, \partial S_{\text{EW}} / \partial \xi = 0.$$